

Characterizing how quantum error correction circuits behave under realistic hardware noise is essential for testing the assumptions that enable scalable fault tolerance. We introduce a protocol to directly estimate the logical Pauli channels (and pure errors) associated with detector regions formed of two or more syndrome extraction gadgets, conditioned on observing a particular parity in the syndrome outcomes. The method is SPAM-robust and we validate this new protocol on a small error-detecting code using Quantinuum H1-1, a trapped-ion device, and demonstrate that several noise diagnostic tests for fault tolerance improve significantly when using noise tailoring and mitigation strategies.

## INTRODUCTION

Logical error rates conditioned on syndrome outcomes have the potential to be a powerful diagnostic tool, enabling noise-aware decoding and validation of threshold-relevant assumptions [1]. To assess whether QEC circuits perform as expected, a characterization protocol should be *direct*, inferring logical-level error models using data with as few additional assumptions as possible, and *syndrome-dependent*, conditional on measurement outcomes obtained at runtime. These properties enable post-selection based on syndrome data [2] and soft decoding within a concatenated code [3–6].

Existing scalable direct methods include averaged circuit eigenvalue sampling (ACES) [7], which does not include noise within mid-circuit measurements, and logical randomized benchmarking [8] which does not integrate syndrome data individually.

## CONDITIONAL ERROR PROBABILITIES OF QEC GADGETS

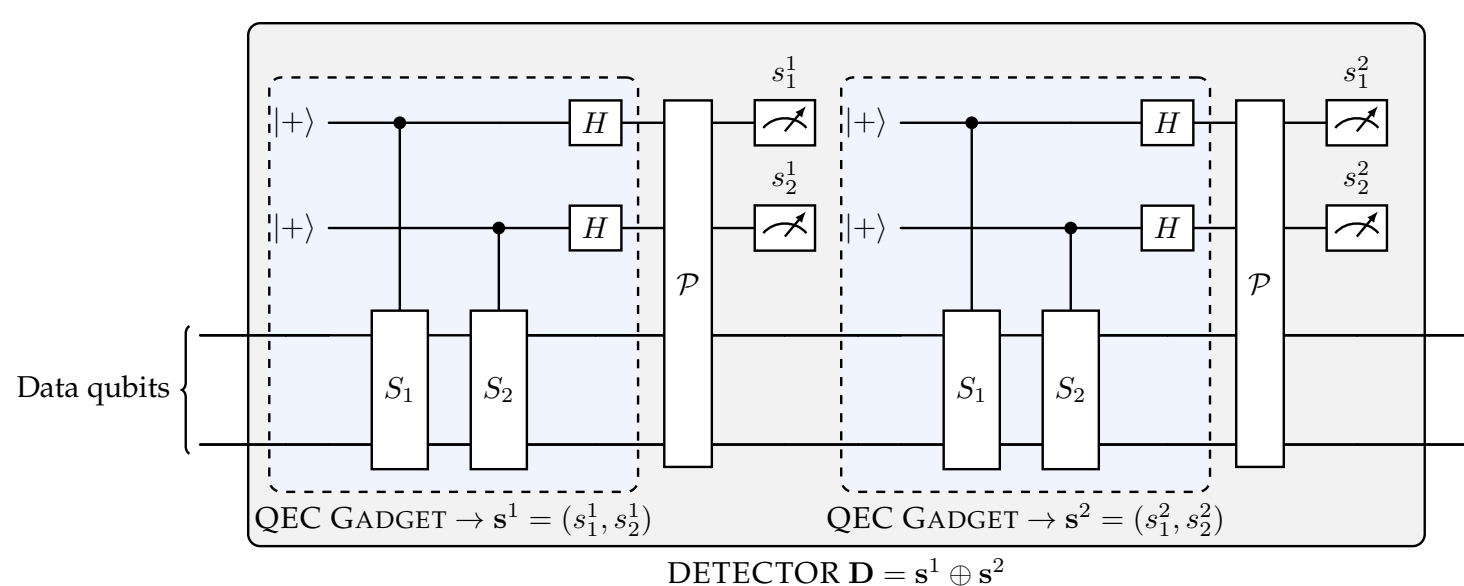


**Figure 1: Independent detector instrument,  $\{\mathcal{E}^D, \mathbf{D}\}$ .** Using circuit level Pauli noise.

Given a circuit level Pauli noise model,  $\mathcal{P}$ , for an arbitrary input state  $\rho$ , a QEC gadget conditioned on syndrome  $s$  can be described by the *quantum instrument*  $\mathcal{I}_s(\rho) = \sum_m \mathcal{P}^{m \oplus s}(\Pi_m \rho \Pi_m)$  where  $\mathcal{P}^m$  is the reduced Pauli error channel acting only on the data qubits which arises from those Pauli errors in  $\mathcal{P}$  that flip the outcome according to  $m$ . To remove input state dependence (as required for SPAM robustness) we can group two consecutive instruments into a detector:

$$\mathcal{E}^D(\rho) := \sum_{s^1} \mathcal{I}_{s^1 \oplus \mathbf{D}} \circ \mathcal{I}_{s^1}(\rho) \quad (1)$$

where the 2<sup>nd</sup> observed syndrome is  $s^2 = s^1 \oplus \mathbf{D}$  and we sum over  $s^1$ . Within a sequence, repeatedly applying a syndrome extraction gadget  $2r$  times results in classical outcomes  $s^1, \dots, s^{2r}$ . From these, by taking the XOR of consecutive syndromes gives detectors. We specifically construct *independent detectors*  $\mathbf{D}_1, \dots, \mathbf{D}_r$  by pairing syndromes disjointly. This guarantees independence between detectors for circuit level Pauli errors.

## NOISE TAILORING TECHNIQUES

To ensure the assumption of stochastic Pauli errors is met on average, we use Pauli frame randomization (PFR) to twirl coherent errors, and regularly swap ancillas with data qubits (leakage protection, LP) to limit the accumulation of leakage errors.

Using H1-1, we compare the detector click rate over time with and without these techniques. The estimated gradient for each dataset is given, where a statistically significant increase in click rate over time is observed without LP. Additionally, PFR can be seen to ensure a consistent click rate between different input states.

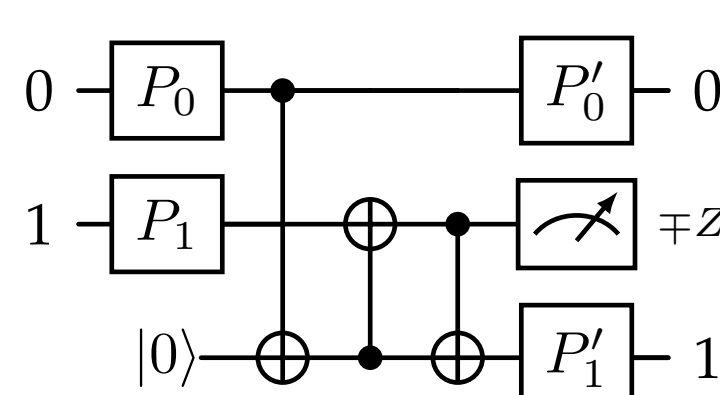


**Figure 2: QEC gadget with LP and PFR for the  $[[2, 1, 1]]$  X-error-detecting code.**

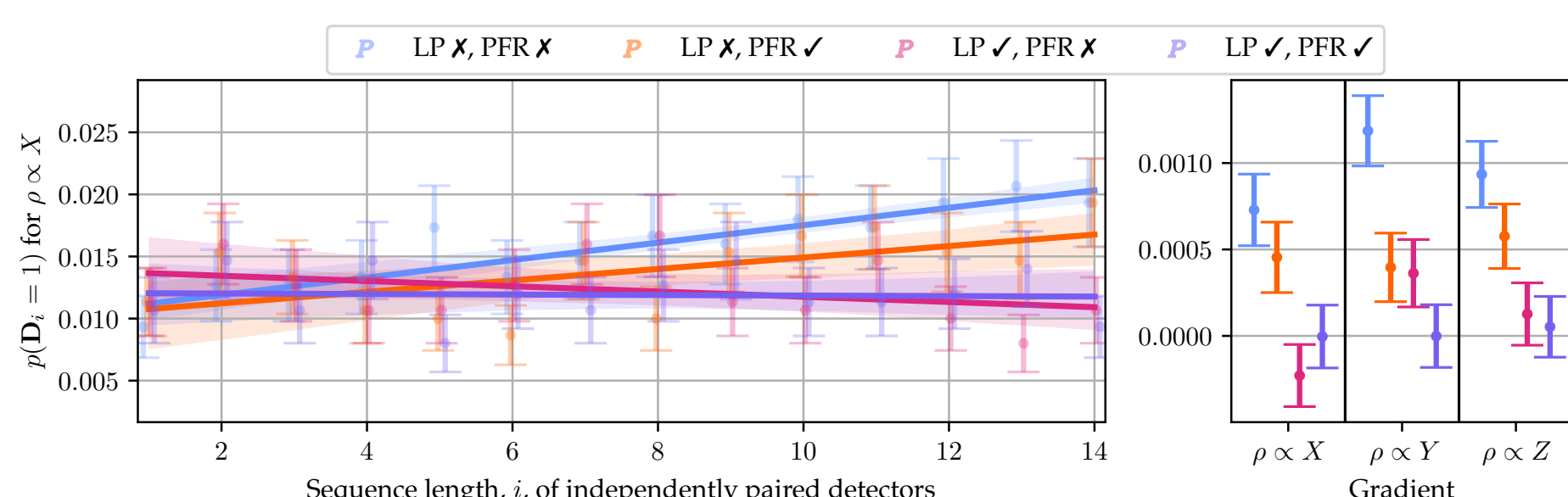


**Figure 3: Mitigation of the effect of leakage errors on detector click rate over sequences for the  $[[2, 1, 1]]$  X-error-detecting code.** A least-squares fit is performed for each input state ( $\approx \rho_{X,Y}$  or  $Z$ ) and we plot one input state ( $\rho_X$ ) across settings.

## LOGICAL SYNDROME-DEPENDENT DETECTOR REGION TOMOGRAPHY

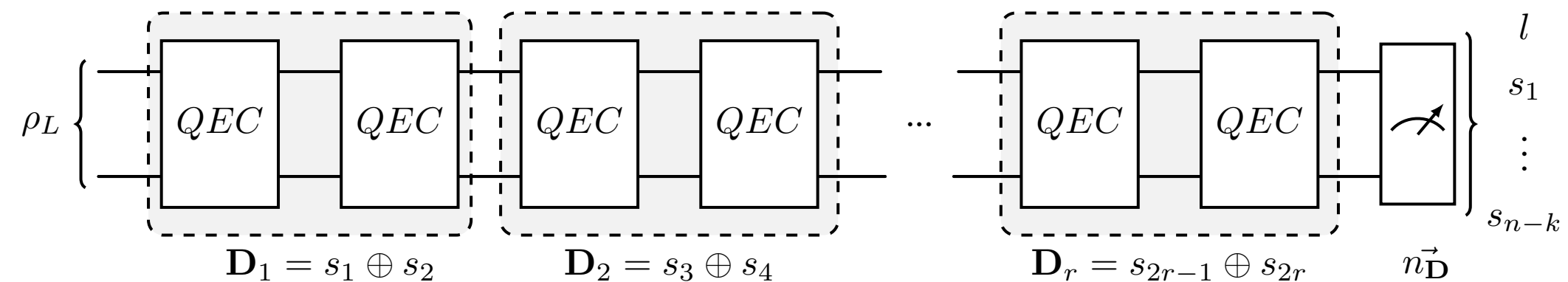


**Figure 4: Forming detectors from independent syndrome pairs in sequences of repeated QEC.**

The following protocol performs a SPAM robust characterization of (Pauli twirled) error channels,  $\mathcal{E}^D$  associated with each outcome,  $\mathbf{D}$ , of independent detectors. For each shot, we:

1. Prepare a +1 eigenstate  $\rho_L$  of a logical Pauli operator  $L$ ,
2. Apply a sequence of independent detectors – recording the total number of each detector outcome  $n_{\mathbf{D}} = (n_{\mathbf{D}=0}, \dots)$ .
3. Destructively measuring  $L$  to obtain  $l \in \{0, 1\}$  and a complete set of stabilizer generators ( $s_1, \dots, s_{n-k}$ ) which allow us to simultaneously measure any logical operator  $Q = LS$  for  $S \in \mathbb{S}$ , the stabilizer group.

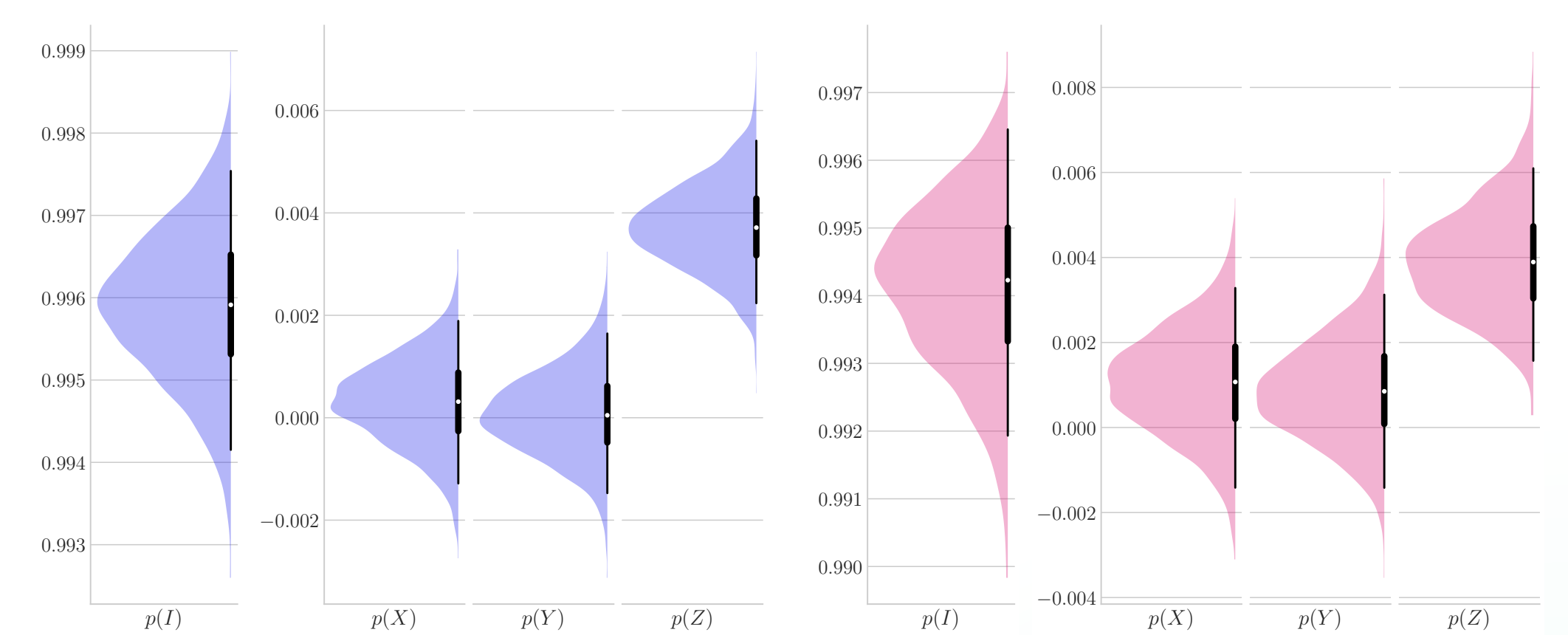
We then fit to the following model to obtain the set of eigenvalues  $\{\lambda_{\mathbf{D}}(Q)\}$  for each detector outcome ( $\mathbf{D} = 0, \dots$ ) and Pauli operator  $Q$ :

$$\mathbb{E}[Q|\mathbf{D}_1, \dots, \mathbf{D}_r] = A_Q \prod_{\mathbf{D}} (\lambda_{\mathbf{D}}(Q))^{n_{\mathbf{D}}}, \quad (2)$$

where  $A_Q$  incorporates the SPAM errors and the product ranges over all unique detector values  $\mathbf{D} \in \{\mathbf{D}_1, \dots, \mathbf{D}_r\}$  that were observed, where each value occurs  $n_{\mathbf{D}}$  times in the given detector sequence. The eigenvalues set can then be inverted to find the probabilities of each Pauli error (up to stabilizers) for each channel,  $\mathcal{E}^D$ .

## RESULTS

Using H1-1, we perform the above protocol for  $[[2, 1, 1]]$  code, with 6 codeblocks in parallel. Figure 5 shows two operational logical channels, where a uniform prior for each eigenvalue has been used. These syndrome-resolved, marginal posterior probability distributions of errors are from a region of two syndrome gadgets with detector outcome  $\mathbf{D} = 0$  that occurs with probability  $p(\mathbf{D} = 0) = 0.9896 \pm 0.0002$  per detector.



**(a)** Logical channel for detector outcome  $\mathbf{D} = 0$  when post-selected on no detectable error  $\mathcal{E}_L^0 / \text{tr}(\mathcal{E}_L^0)$ . **(b)** Logical channel for detector outcome  $\mathbf{D} = 0$  when followed by an ideal decoder  $(\mathcal{E}_L^0 + \mathcal{E}_X^0) / \text{tr}(\mathcal{E}_L^0 + \mathcal{E}_X^0)$ .

**Figure 5: Logical detector channel estimation for the  $[[2, 1, 1]]$  X-error-detecting code on H1-1 using Bayesian statistics.** For both  $\mathbf{D} \in \{0, 1\}$  we can write  $\mathcal{E}^D(\rho) = \mathcal{E}_L^D(\rho) + IX\mathcal{E}_X^D(\rho)IX$  where  $\mathcal{E}_L^D$  &  $\mathcal{E}_X^D$  are (sub-normalized) logical Pauli channels and  $IX$  is the pure error of the code. We then derive and plot two operational channels.

These results show that when the  $[[2, 1, 1]]$  code is used as a low-level code in a concatenated scheme, it can not only provide a long-lived QEC memory [9], but with suitable noise tailoring, it can do so with improved logical fidelities.

## OUTLOOK

Future directions include applying our scheme directly to higher distance codes and, within concatenated schemes, using a lower level detector characterization for soft decoding at higher levels. Quantifying the mismatch between logical channels extrapolated from physical level benchmarking and direct methods is key for better QEC resource estimation.