

Motivated by benchmarking quantum computers, and certifying non-classicality in quantum physics, we have the following two aims:

- (1) To quantify the degree to which a quantum channel deviates from being separable in a form that can be estimated efficiently and robustly,
- (2) To demonstrate an application of this approach by deriving an information-disturbance relation that can be efficiently and robustly verified.

UNITARITY AND RANDOMIZED BENCHMARKING

Our work exploits recent randomized benchmarking techniques [1–4] that provide additional information on the average gate infidelity $r(\mathcal{E})$ for noise channels. The *unitarity* $u(\mathcal{E})$ of a quantum channel $\mathcal{E}$ is defined as

$$u(\mathcal{E}) := \frac{d}{d-1} \int d\psi \operatorname{tr} \left[\mathcal{E}(\psi - \frac{\mathbb{1}}{d})^2 \right], \quad (1)$$

where the integration is with respect to the Haar measure [1]. It measures how far a quantum channel is from being a unitary channel and can be estimated by an efficient and SPAM-robust protocol.

SUBUNITARITIES OF QUANTUM CHANNELS

We generalize the concept of unitarity to the *subunitarities* of bipartite channels, $\mathcal{E}_{AB}: \mathcal{B}(\mathcal{H}_A \otimes \mathcal{H}_B) \rightarrow \mathcal{B}(\mathcal{H}_A \otimes \mathcal{H}_B)$.

Definition 1. The sub-unitarity $u_{A \rightarrow A}$ is defined as

$$u_{A \rightarrow A}(\mathcal{E}_{AB}) := u(\mathcal{E}_A), \quad (2)$$

where $\mathcal{E}_A(\rho) := \operatorname{tr}_B \circ \mathcal{E}_{AB}(\rho \otimes \mathbb{1}_B/d_B)$.

This measure quantifies the degree to which coherent quantum information is maintained in a subsystem under a bipartite channel.

We define 8 other measures, $u_{i \rightarrow j}$. For subsystem A define an orthonormal basis of operators $a_\mu = (a_0 = \mathbb{1}_A/\sqrt{d_A}, a_i)$, and similarly for B . Through vectorization this complete basis provides the following decomposition of the Liouville representation $\mathcal{L}(\mathcal{E}_{AB})$:

$$\mathcal{L}(\mathcal{E}_{AB}) = \begin{matrix} \langle a_0 \otimes b_0 | \\ \langle a_{i_1} \otimes b_0 | \\ \langle a_{i_1} \otimes b_{i_2} | \\ \langle a_0 \otimes b_{i_2} | \end{matrix} \begin{pmatrix} |a_0 \otimes b_0\rangle & |a_{j_1} \otimes b_0\rangle & |a_{j_1} \otimes b_{j_2}\rangle & |a_0 \otimes b_{j_2}\rangle \\ 1 & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{x}_{A \rightarrow A} & T_{A \rightarrow A} & T_{AB \rightarrow A} & T_{B \rightarrow A} \\ \mathbf{x}_{AB \rightarrow AB} & T_{A \rightarrow AB} & T_{AB \rightarrow AB} & T_{B \rightarrow AB} \\ \mathbf{x}_{B \rightarrow B} & T_{A \rightarrow B} & T_{AB \rightarrow B} & T_{B \rightarrow B} \end{pmatrix}$$

Definition 2. For any bipartite quantum channel $\mathcal{E}_{AB}$ the sub-unitarity $u_{X \rightarrow Y}$ of the channel is defined as:

$$u_{X \rightarrow Y}(\mathcal{E}_{AB}) := \alpha_X \operatorname{tr} [T_{X \rightarrow Y}^\dagger T_{X \rightarrow Y}], \quad (3)$$

for any $X, Y \in \{A, B, AB\}$ and with $\alpha_A = 1/(d_A^2 - 1)$, $\alpha_B = 1/(d_B^2 - 1)$ and $\alpha_{AB} = \alpha_A \alpha_B$.

These measures quantify the transfer of coherent quantum information between different subsystems.

ROBUST INFORMATION-DISTURBANCE RELATION

The following result relates directly to quantum incompatibility [5].

Theorem 1 (Information-disturbance relation). For any channel $\mathcal{E}_{X \rightarrow A}$ and an associated complimentary channel $\mathcal{E}_{X \rightarrow B}$

$$u(\mathcal{E}_{X \rightarrow A}) + u(\mathcal{E}_{X \rightarrow B}) \leq 1, \quad (4)$$

where $\mathcal{E}_{X \rightarrow A} := \operatorname{tr}_B \circ \mathcal{E}_{X \rightarrow AB}$ and $\mathcal{E}_{X \rightarrow B} := \operatorname{tr}_A \circ \mathcal{E}_{X \rightarrow AB}$ for any isometry $\mathcal{E}_{X \rightarrow AB}$.

For a pair of complimentary channels, this bound limits how coherent quantum information can be transferred into each output system.

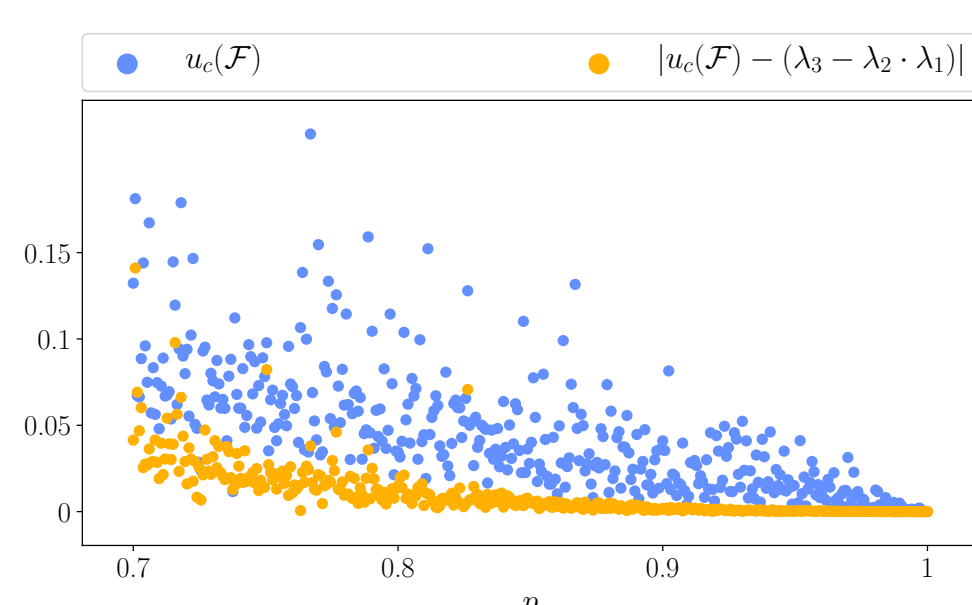


Figure 1: SPAM error robust estimation of u_c for generic quantum channels. For a 2 qubit simulation, we show $|u_c - C|$ over p , where $\mathcal{F} = p\mathcal{E}_A \otimes \mathcal{E}_B + (1-p)\mathcal{G}$.

CORRELATED UNITARITY

Definition 3. The correlated-unitarity of a bipartite channel $\mathcal{E}_{AB}$ is

$$u_c(\mathcal{E}_{AB}) := u_{AB \rightarrow AB}(\mathcal{E}_{AB}) - u_{A \rightarrow A}(\mathcal{E}_{AB}) u_{B \rightarrow B}(\mathcal{E}_{AB}) \quad (5)$$

where $u_{i \rightarrow i}(\mathcal{E}_{AB})$ are subunitarities of the channel.

This channel correlation measure parallels the covariance of two random variables and satisfies a range of operational properties such as invariance under local changes of bases, and is bounded $u_c(\mathcal{E}_{AB}) \leq 1$. Correlated unitarity allows us to certify non-classical features of a channel. In particular, over the set of *separable* bipartite quantum channels, $\mathcal{E}_S := \sum_i p_i \mathcal{E}_{A,i} \otimes \mathcal{E}_{B,i}$, it is strictly bounded away from 1.

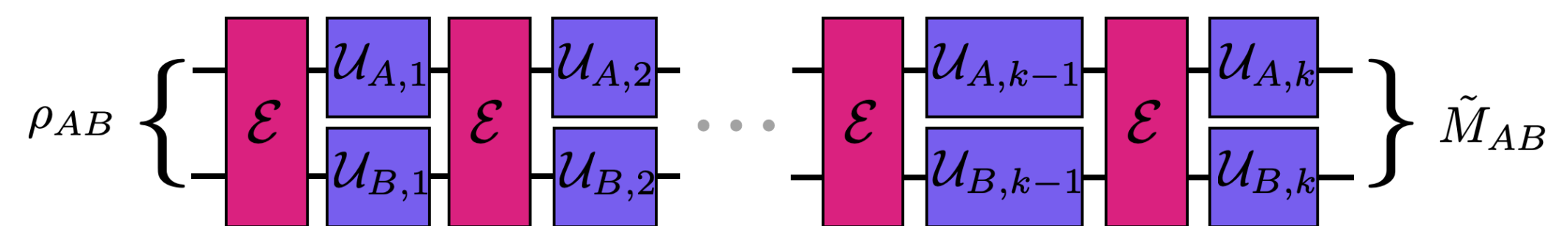
Theorem 2 (Witness of non-separability). Given a bipartite quantum system AB with subsystems A, B of dimensions d_A and d_B respectively, for a bipartite separable channel $\mathcal{E}_S$ we have that

$$u_c(\mathcal{E}_S) \leq C(d_A, d_B) \leq \frac{17}{24} < 1, \quad (6)$$

which is maximised for $C(3, 3) = \frac{17}{24}$. The 2 qubit case yields $C(2, 2) = \frac{7}{12}$.

SUBSYSTEM COHERENCE MEASURES THROUGH RB

We now consider coherent correlations in the averaged noise channel, $\mathcal{E}$, over a computational gate-set.



We perform a novel unitarity benchmarking scheme, randomly sampling Clifford group unitaries on each subsystem *simultaneously* (E.g. $\mathcal{U}_{AB,i} = \mathcal{U}_{A,i_1} \otimes \mathcal{U}_{B,i_2}$). Over many random sequences of length k , the average value of some observable squared, $\mathbb{E}_s[(m)^2]$, will behave as

$$\mathbb{E}_s[m(s)^2] = c_{00} + c_{01}\lambda_1^{k-1} + c_{10}\lambda_2^{k-1} + c_{11}\lambda_3^{k-1}, \quad (7)$$

where $(\lambda_1, \lambda_2, \lambda_3)$ are the eigenvalues of the matrix of subunitarities for the average gate noise, $\mathcal{E}$:

$$\mathcal{S} = \begin{pmatrix} u_{A \rightarrow A}(\mathcal{E}) & \frac{1}{\sqrt{\alpha_B}} u_{AB \rightarrow A}(\mathcal{E}) & \frac{\sqrt{\alpha_A}}{\sqrt{\alpha_B}} u_{B \rightarrow A}(\mathcal{E}) \\ \sqrt{\alpha_B} u_{A \rightarrow AB}(\mathcal{E}) & u_{AB \rightarrow AB}(\mathcal{E}) & \sqrt{\alpha_A} u_{B \rightarrow AB}(\mathcal{E}) \\ \frac{\sqrt{\alpha_B}}{\sqrt{\alpha_A}} u_{A \rightarrow B}(\mathcal{E}) & \frac{1}{\sqrt{\alpha_A}} u_{AB \rightarrow B}(\mathcal{E}) & u_{B \rightarrow B}(\mathcal{E}) \end{pmatrix}, \quad (8)$$

with $\alpha_X = 1/(d_X^2 - 1)$, and where the constants $c_{00}, \dots, c_{11}$ describe the SPAM errors. Therefore, the subunitarities arise naturally in the context of this benchmarking.

ROBUST ESTIMATION OF CORRELATIONS

Given $\lambda_1 \geq \lambda_2 \geq \lambda_3$, we compute a SPAM robust measure $C := |\lambda_3 - \lambda_1 \cdot \lambda_2|$. On separable channels $C = u_c$ and $C = 0$ for product channels. It is therefore a useful measure of correlations in noise channels (see Figure 1).

OUTLOOK

- Exact for Pauli channels, so use additional Randomized Compiling [6] to estimate for non-separable channels?
- Benchmarking defined coherence measures on real world devices.
- Applications of Thm 2 to certify coherent quantum information transfer between sub-components of a quantum circuit. (In preparation) *Certifying Quantum No-Go Theorems*, M. Girling, C. Cîrstoiu, D. Jennings.